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AN EXAMINATION OF THE NASDAQ 100 FUTURES CONTRACT USING ULTRA HIGH FREQUENCY DATA

[a]Fathi Abid, [b]Lotfi Trabelsi*
[a] *Faculty of Business and Economics of Sfax, University of Sfax, Tunisia. MO.DE.S.FI, Tunisia.*
[b] *Institute of the High Commercial Studies of Sfax, University of Sfax, Tunisia. MO.DE.S.FI, Tunisia.*

A B S T R A C T

This paper conducts a study on a high frequency data of futures index contracts after the examination of the Nasdaq 100 to investigate the effects of price duration in trading process. To achieve this prospect, we extend the Engle and Russell (1998) model, which divides the intensity effect into liquidity and information components by including additional microstructural variables. Examining tick by tick data of Nasdaq 100 futures index futures; we find that the time duration between transactions exerts a considerable influence on price changes. Additionally, the time can be modelled in combination with variable microstructure. This evidence suggests that managing both time, trading volume and microstructural variables are important aspects of trading in the index futures markets.

Keywords: Index futures contracts, market microstructure, price impact, ultra-high frequency data, Autoregressive conditional duration.

INTRODUCTION

In this paper, we deal with the price effects of duration within a microstructure model of price discovery for an index future contract, the Nasdaq 100. For instance, duration is defined as the time elapsed between consecutive trades. The importance of time in price discovery emerges clearly in Easely and O'Hara (1992) model. In this model, the information flow is not continuous because informed traders choose not to trade from time to time. In empirical studies duration is respectively considered not only as a measure of trading intensity but also a measure of liquidity and a measure of risk. In all the studies of E.g.Jasiak and Ghysels (1998), Engle and Russell (1997), Engle and Russell (1998), Engle (2000), Renault and Werker (2011), Manganelli (2002) and Spierdijk (2004), they consider duration as a measure of trading intensity. These studies show that duration is inversely proportional to the expected return variance. In this field Engle (2000) shows that variation in duration and variation in return variance are linked to the same news events, whereas in the studies of Gouriéroux, Jasiak and Le Fol (1999), Dufour and Engle (2000), Engle and Lange (2001), Engle and Lunde (2004) duration is considered as a measure of liquidity.

* Corresponding Author:
Email: trabelsil@yahoo.fr

The general result in these studies show that liquidity is an important determinant of bid-ask spread, and that it can be estimated with trading activity-based measures. On the one hand, Dufour and Engle (2000) show that informed traders are more active during periods when the number of informed transactions can be maximised. On the other hand, Renault and Werker (2011), and Ghysels, Gourièroux, and Jasiak (2004) duration is considered as a measure of risk.

With the exception of the econometric analysis of transition Lancaster (1990), the time series models ensure that the time interval between the observations is equal over time, which is not the case with high frequency data.

To provide a structure for the econometric analysis of irregularly spaced data, Engle and Russell (1997) proposed the autoregressive conditional duration (ACD) model combining hazard function and ARCH models. In these researches, the authors model durations between successive market transactions of a stock, rather than adopting the traditional perspective of examining the volatility of the price process. Since the presentation of this model, empirical analysis of time between events listing have developed rapidly in different directions and have incorporated more in the assumptions of microstructure. A common subject of recent theoretical models of microstructure is that the time between various events such as the transaction whose information affects both the behavior of actors and the process of price formation.

Therefore, the first studies to use the duration as a random variable has been devoted to the analysis of the occurrence of transactions. Besides Engle and Russell (1998), and Shphard Rydberg (1998) analyze the arrivals of orders using a Cox process. Hautsch (1999) examines the time between transactions using a semi-parametric model of chance. Furthermore, Gourieroux, Le Fol and Jasiak (1999) introduce duration-based measures of activity; they define different classes of duration as the duration weighted by volumes. This illustrates the dependence between durations, volumes and price Time or explicitly the transaction duration (the time difference between two consecutive transactions) is considered to be a measure of trading intensity, Engle and Russell, (1997,1998); Engle (2000); Grammig and Wellner (2002); Renault and Werker (2011) and Manganelli (2005). All these studies show that duration is inversely proportional to the expected return variance. For example, Engle (2000) provides empirical evidence showing that variations in duration and variations in returns variance are linked to the same news events. Additionally, in some analysis, duration is considered to be a measure of liquidity, which is noticed in Dufour and Engle (2000), Engle and Lange (2001) and Engle and Lunde (2003). Duration is also documented as a natural measure of the speed by which prices incorporate new information. Dufour and Engle (2000) show that informed traders choose to trade in periods that maximize the number of informed transactions. Duration is also considered as measuring trading risk; see Gouriéoux et al. (1999), Renault and Werker (2011), and Ghysels et al. (2004). In these studies, duration is modelled as a process capturing the risk associated with trading under both price and time uncertainty. For example, in Renault and Werker (2011), duration is split into a deterministic component with transient effects on returns, and a stochastic component with permanent effects on returns variances.

The "time for money", according to Engle and Russell (1998), can be considered as an inverse function of the volatility model. Indeed, a "price-duration" measures the time required for a price change by one person. More time needed to obtain a price change is long, the market is less volatile. This measure of volatility is considered by many researchers as a GARCH model fits the data at high frequency. In addition to the development of ACD model, other models have been developed to describe the volatility. Jasiak and Ghysels (1998) developed a bivariate model ACD-GARCH which takes into account the interactions between the volatility of returns and past durations. Veredas and Bauwens (1999) formulate a variance of this model: the SCD (Stochastic Conditional Duration) based on stochastic volatility models. Ghysels, Gourieroux and Jasiak (1999) determine a stochastic volatility model with SVD (Stochastic Volatility Duration) which introduces dependencies in the durations.

These different volatility models will allow better analysis of the various intraday processes. Among the various interests of such models, Prigent, Renault and Scaillet (1991) include the adjustment of hedging portfolio. These models can also be very useful to test hypotheses of microstructure, Engle and Russell (1998), Bauwens and Giot (1999) as well as to assess the liquidity offered as Engle and Lange (1998).

HIGH FREQUENCY DATA AND DURATION MODELS

1. The Autoregressive Conditional Duration models:

The main characteristic of high frequency data is the fact that they are irregularly time-spaced. Therefore, they are statistically viewed as point processes. A point process is "a special kind of stochastic process, which generates a random collection of points on the time axis", like in the study of Bauwens and Giot (2001, p.67) . A high-frequency financial dataset contains a collection of financial events such as trades, quotes, etc. and, consequently, the times of these events represent the arrival times of the point process. When different characteristics are associated with an event (such as, for example, the price and the volume associated with a trade), they are called marks, and the double sequence of arrival times and marks is called a marked point process. Point processes are widely used in fields such as queuing theory and neuroscience but have attracted great interest in high-frequency finance over the last few years after Engle (2000) used them as a framework for the analysis of the trading process and of market behavior.

1.1 The ACD models: The ACD is a type of dependent point process particularly suited for modeling characteristics of duration series such as clustering and over dispersion. This parameterization is the most easily expressed in terms of the waiting times between events.

Let N be the number of events in prices that occurs randomly during the session. These N events are denoted i=1,..,N observed from the "X" to the last event.

Let $X_i = t_i - t_{i-1}$ be the interval of time between event arrivals, called duration.

ψ_i is the deterministic component which is the conditional expectation of the duration given the past arrival times:

ε_i is the random component;

The multiplicative relationship can be written as follows:

$$X_i = \psi_i * \varepsilon_i$$

Engle and Russell (1998) assume that this component is identically and independently distributed.

ε_t ~i.i.d

This means that the conditional expectation of the time (deterministic component) must capture the dependence of duration in time.

Engle and Russell (1998) suggest and apply linear parameterizations for the expectation given by:

$$\psi_i = \omega + \sum_{j=1}^{p} \alpha_j * X_{i-j} + \sum_{k=1}^{q} \beta_k * \psi_{i-k}$$

Since the conditional expectation of the duration depends on p lags of the duration and q lags of the expected duration this is termed an ACD (p, q) model.

1.2 The likelihood function: The hypothesis concerning the specification of the conditional density of the time allows us to estimate the log-likelihood function as follows:

$$L(X_1,\dots,X_{N(T)};\theta) = \sum_{i=1}^{N(T)} \text{Log } f(X_i / X_1,\dots,X_{i-1};\theta) = \psi_i$$

and $\psi_i = \int_0^{N(T)} X_i \,.\, g(X_i / X_1,\dots\dots,X_{i-1}) d_i$

Where $g(X_i / X_1,\dots\dots,X_{i-1})$ indicates the conditional density function of the duration, while is a hazard. If we divide the duration by the conditional expectation function of duration, the residuals are theoretically independent and identically distributed. In this way the likelihood function becomes:

$$L(X_1,\dots\dots,X_{N(T)};\theta) = \sum_{i=1}^{N(T)} \text{Log } f(\varepsilon_i;\theta) =$$

$$\sum_{i=1}^{N(T)} Log\left[\lambda(t / N(t), t_1,\dots\dots,t_{N(T)} * e^{(-\varepsilon_i)}\right]$$

The log-likelihood function will then depend on the specification and modeling of the conditional intensity. According to the chosen specification, it is possible to find the conditional intensity function of the transactions. For this, it is necessary to introduce the concept of hazard function. At time *t*, it is interpreted as the probability of finding of a transaction at time *t* knowing that there were no transactions for a certain period. The hazard function is the ratio of the density function's length and the survival function of the same duration. This later characterizes the probability of remaining in the initial state for at least y time units.

Let $\lambda_0(t)$ the basic hazard function for standardized lengths, $P_0(t)$ the corresponding density, and $S_0(t)$ the associated survivor function: $\lambda_0(t) = \dfrac{P_0(t)}{S_0(t)}$

regardless of the length distributions (standardized or not), we can write the conditional intensity of events as follows:

$$\lambda(t / N(t), t_1,\dots\dots,t_{N(T)}) = \lambda_0(t)\left\{\frac{t - t_{N(t)}}{\psi_{N(t)+1}}\right\}\frac{1}{\psi_{N(t)+1}}$$

The shape of the hazard function depends on the choice of the distribution followed by the different durations.

1.3 The basic specification: The basic reference for ACD (p, q) model is Engle and Russell (1998), they specify the duration X_i as:

$$X_i = \psi_i * \varepsilon_i$$

$$\psi_i = \omega + \sum_{j=1}^{p} \alpha_j * X_{i-j} + \sum_{k=1}^{q} \beta_k * \psi_{i-k}$$

Constraints sign of the coefficients α and β depend on the distribution followed, while the coefficient ω is always strictly positive.

This shows that the first two moments are time-varying because they are calculated from the conditional expectation of times and this regardless of the hazard function. Thus, we have a conditional expectation equal to ψ_i. The unconditional durations, denoted μ, can be written according to Engle and Russell (1997):

$$E(x_i) = \mu = \frac{\omega}{1-\alpha-\beta}$$

Engle and Russell (1997) demonstrate this result, and indicate that the necessary and sufficient condition to validate the existence of the medium is the presence of all the roots of the polynomial characteristic associated outside the circle of unit root.

The conditional variance is equal to ψ_i^2 while the unconditional variance is written as follows:

$$\sigma^2 = \mu^2\left(\frac{1-2\alpha\beta-\beta^2}{1-(\alpha+\beta)-\alpha^2}\right)$$

It is clearly noticed that when the coefficient is positive, the unconditional standard deviation is above the average, which means that there is too much dispersion.

2. The Exponential Autoregressive Conditional Duration model: The first model proposed by ACD literature is the EACD model, Exponential Autoregressive Conditional Duration, Engle and Russell (1998). However, we make the assumption which the durations are conditionally exponential where ε_i (the standardized durations) follow an exponential distribution.

Thus the following two equations characterize the EACD model: $X_i = \psi_i * \varepsilon_i$ where ε_i an exponential distribution.

$$\psi_i = \omega + \sum_{j=1}^{p} \alpha_j * \tilde{X}_{i-j} + \sum_{k=1}^{q} \beta_k * \psi_{i-k}$$

The EACD (1.1) model is presented as follows:

$$\psi_i = \omega + \alpha_j * \tilde{X}_{i-j} + \beta * \psi_{i-1}$$

Sign constraints are: the coefficients α *and* β are nonnegative and the sum of these coefficients must be less than or equal to unity and the coefficient ω remains strictly positive.

Thus the model reflects the autoregressive duration. Due to the positive or zero sign of the α and β coefficients, a short (long) past period leads to the appearance of a short (long) duration. The phenomenon of clustering of transactions is well explained by the model.

The conditional expectation of the duration is calculated as: $E(X_i / I_{i-1}) = \phi_i * 1 = \phi_i$

Where the past information $I_{i-1} = (X_{i-1}, \psi_{i-1})$

The conditional expectation of the duration is equal to:

$$\phi_i = \psi_i \text{ and } \varepsilon_i = \left(\frac{X_i}{\psi_i}\right)$$

In this case, the hazard function associated with the standardized duration is equal to unity. Therefore, the conditional intensity function is estimated as follows:

$$\lambda(t / X_1, \ldots, X_{N(T)}) = \frac{1}{\psi_{N(t)+1}}$$

However, this density function is the hazard function of duration. Consequently, the density function of the duration is expressed as follows:

$$f(X_i / X_{i-1}) = \lambda(t / X_1, \ldots, X_{i-1}) * e^{(-\varepsilon_i)} = \frac{e^{\frac{X_i}{\psi_i}}}{\psi}$$

Finally, the Log likelihood function can be written as:

$$L(X_1, \ldots, X_{N(T)}) = \sum_{i=1}^{N(T)} Log(\psi_i) - \left(\frac{X_i}{\psi_i}\right)$$

3. The Weibull Autoregressive Conditional Duration model: Engle and Russell (1998) also propose a Weibull (1, γ) distribution for the standardized durations, WACD (Weibull Conditionnal Autoregressive Duration).

The greater flexibility of this distribution from the exponential one is the fact that the conditional intensity function is a function with one parameter; its role is to determine the evolution of increasing or decreasing hazard function.

Thus the following two equations of the model WACD: $X_i = \psi_i * \varepsilon_i$ where ε_i Weibull (1, γ) distribution.

$$\psi_i = \omega + \sum_{j=1}^{p} \alpha_j * \tilde{X}_{i-j} + \sum_{k=1}^{q} \beta_k * \psi_{i-k}$$

The WACD (1.1) model is presented as follows:

$$\psi_i = \omega + \alpha_j * \tilde{X}_{i-j} + \beta * \psi_{i-1}$$

Sign constraints are identical to that of the EACD model. The conditional expectation of the duration between events is calculated as the mean of a Weibull distribution (1, γ) is $\Gamma(1+1/\gamma)$ as follows:

$$E(X_i / I_{i-1}) = \phi_i * \Gamma[1+1/\gamma] = \psi_i$$

We deduce the following equality:

$$X_i = \frac{\psi_i}{\Gamma[1+1/\gamma]} * \varepsilon_i$$

The standardized durations can be expressed as follows:

$$\varepsilon_i = \frac{X_i * \Gamma[1+1/\gamma]}{\psi_i}$$

Given the hazard function associated with the standardized duration, the conditional intensity function can be estimated by using the formula:

$$\lambda(t / t_1,\ldots,t_{N(T)}, N(t)) = \left(\frac{\Gamma[1+1/\gamma]}{\psi_{N(t)+1}}\right)^{\gamma} * \left(t - t_{N(t)}\right)^{\gamma-1} * \gamma$$

From the estimated duration, the standardized density function of the time between two prices events will be expressed as follows:

$$f\left(X_i / I_{i-1}\right) = \lambda(t / X_1,\ldots,X_{i-1}) * e^{(-\varepsilon_i)^{\gamma}}$$

That is:

$$f\left(X_i / I_{i-1}\right) = \frac{\gamma}{X_i} * \left(\frac{\Gamma[1+1/\gamma] * X_i}{\psi_i}\right)^{\gamma} * e^{\left(\frac{-\Gamma[1+1/\gamma] * X_i}{\psi_i}\right)^{\gamma}}$$

Then the function of Log Likelihood is estimated as follows:

$$L(X_1,\ldots,X_{N(T)}) =$$

$$\sum_{i=1}^{N(T)} \text{Log}\left(\frac{\gamma}{X_i}\right) + \gamma * \text{Log}\left(\frac{\Gamma(1+1/\gamma) * X_i}{\psi_i}\right) - \left(\frac{\Gamma(1+1/\gamma) * X_i}{\psi_i}\right)^{\gamma}$$

When γ <1, the hazard function is decreasing, which means that the probability of having a long duration is low, conversely, if γ > 1, then the probability of a longer extended trading between events becomes higher.

When γ = 1, we find the special case where the Weibull distribution is an exponential one.

4. Association between ACD models and market microstructure variables: The original ACD model, Engle and Russell (1998) have led to several extensions. Some authors have sought to improve the explanation of the model by incorporating additional variables, allowing them the same opportunity to test hypotheses suggested by the theoretical literature of microstructure.

As GARCH models, Lamoureux and Lastrapes (1990), Najand and Yung (1991), Foster (1995) the authors interested in ACD models incorporate exogenous variables in the time process to improve prediction and to explain the autoregressive character of the time between transactions.

4.1 The trading intensity: It is defined as the number of transactions during a price duration, divided by the value of this duration. By introducing this exogenous variable in the returns process the number of transactions that occurs within a time period has an influence on prices. If we consider an illiquid share because of its low number of transactions, we can assume that volatility is higher since each transaction has an influence on the price level. Since there is a little activity, the events of prices should be poor and therefore the duration will be long.

Conversely, liquid shares are the subject of many transactions; therefore, the occurrence of significant price changes takes on a frequent basis. The "price-times" are then shorter.

In the literature, which is expanding in recent years, the variables of activities are used as proxy for the arrival of market information. Thus, Lamoureux and Lastrapes (1990) or Najand and Yung (1991) introduce the level of activity as an explanatory variable in the equation of the conditional variance in a GARCH model. The authors justify the fact that the information conveyed by trading volumes influences the level of volatility.

According to Foster (1995), using the generalized method of moments, the activity and volatility appear to be rather directed by the same variable: the rate of information diffusion. In the same way, this reflection was joined in some analysis of Blume, Easley and O'Hara (1992), in which the volume would provide more information on signal quality of market participants.

The Easley and O'Hara (1992) model predicted that the number of transactions involve the price discovery process across information that detects when the transaction is concentrated, and the bid-ask spread should be frequently revised at the quoted price. In contrast the model of Admati and Pfleiderer (1988) predicted that the number of transactions does not affect the intensity of price.

The results obtained by Engle and Russell (1997) tend to confirm the assumptions cited by Easley and O'Hara (1992) as the existing relationship between "price-duration" and the transaction rate increased significantly negative. This means that the time between two transactions is a short price. Bauwens and Giot (1999) confirm this negative relationship by analyzing the display spread on the NYSE.

4.2 The Bid-Ask Spread: All of Engle and Russell (1998), Engle and Lange (1997) as well as Bauwens and Giot (1999) propose to integrate the nominal spread. While the change of the BAS does not affect the modelling process events in prices since the various authors hold the middle of the BAS. However, a wide BAS means that the asset has a higher volatility: there is a greater degree of uncertainty about the fundamental value, or equilibrium value of the assets. Thus, a wide BAS should generate a greater number of significant price changes, so these transactions should take place more frequently.

This approach corroborates the findings of Engle and Russell (1998), Bauwens and Giot (1999) as well as those of Engle and Lange (1997).These authors, that model respectively EACD (2.2) and WACD (1.1), obtained very significant results, the width of the BAS is negatively correlated with his survivor function: this means that an asset with a small BAS price (wide), allow a long (low) duration between price changes. However, according to Engle and Russell (1998), the presence of BAS variable would not seem to influence the value of the coefficients but the BAS appear to be due to the transaction rate.

4.3 The mean size of orders: Engle and Russell (1998) also introduce the number of assets traded per transaction, which is also significant in the regression, however, it should be noted that the authors use this new variable in a model where two exogenous variables, the BAS and the transaction rate, are already present.

From another part, Easley and O'Hara (1992) consider that in the market there are two kinds of investors: the initiated and the uninitiated. Uninformed agents, also called "liquidity traders" buy and sell for liquidity reasons. For instance, they believe that the observed level can be decomposed into two kinds of volume. There is a "normal volume" which is the level of liquidity through the financial asset; it is the result of trade conducted on the initiative of the uninformed agents. The difference between the observed level and the "normal size" determines the "unexpected volume" corresponding to transactions by informed investors. Thus, the probability of negotiating with stakeholders increases when the volume initiates abnormal increases, which causes a change in the BAS of the market maker faced with the problem of adverse selection.

Furthermore, Bauwens and Giot (1997, 1999) analyze the NASDAQ and the NYSE relation to the abnormal amount and don't get significant results on the NASDAQ. One explanation advanced by the authors is the fact that they should take into account the daily seasonality as Monday effect, Jain and Joh (1988), Foster and Viswanathan (1990). The results obtained on the NYSE, however, corroborate the analysis of Easley and O'Hara (1992).

In fact, the negative relationship between the duration and the size of the transaction can also be explained without using the theory of the price-volume relationship. A transaction results at a large decrease in the depth of the order book, as the probability of receiving another increase of limit price. It should be noted that some authors believe that the size of orders is not significant in the formation process of price discovery, this is explained by the fact that informed investors prefer to move in the market face covered by running small orders to take advantage as long as possible on their inside information. This is one of the foundations of the analysis of Jones, Kaul and Lipson (1994) that demonstrate that the occurrence of transactions which has a role in the volatility, as endogenous variable to the number of transactions.

4.4 The imbalance: On the one hand, Bauwens and Giot (1997) analyze the process of displaying the BAS on the NASDAQ, governed by market prices. On the other hand, Glosten and Milgrom (1985) construct a model of information in which investors and market makers haven't the same information about the fundamental value of the asset they exchange. We can deduce that If there were as uninformed investors in the market, the BAS would be reduced and purchase transaction volumes should be equivalent to seller transaction volume. As the situation differs in the presence of inside investors, they will buy or sell based on their information. Recognizing this, the market maker revises the BAS. According to Bauwens and Giot (2000), modelling the time between BAS may explain variations in liquidity during the session since this model takes into account the past behaviour of market makers and the imbalance of the past market.

DATA SIMULATION ON ACD DURATION MODELS

To illustrate the ACD process, we generated 13,554 observations from the ACD model (1.1) using two different distributions of th ε_i "exponential" and "Weibull". The data relate to transactions on the futures contract on the NASDAQ 100 index with a date maturity from 1/8/1998 to 9/17/1998. This U.S indexes futures contract is traded in Chicago, he was selected primarily for its liquidity. Futures contracts on derivatives are introduced by CBOT in 1975 reflecting the enormous growth of the spot market. This type of derivative has allowed investors to hedge against fluctuations in the index and speculating on their future direction. Each time, four expiration dates are available to the investor: March, June, September and December.

Before beginning the analysis of these data, it is crucial to understand the intra-day system operations in the

Chicago. In this index contracts market, the activities of regular transactions begin at 7: 20 am and end at 2:00 pm. However, market participants can trade during the night. The intraday Chicago Stock Exchange database contains date, time recording, bid, ask, volume of contracts traded and open interest. To filter our data, we removed invalid transactions and daily inter-times to select only those transactions that are inside the regular session. Given the presence of multiple concurrent transactions, we chose to replace them with the weighted average prices calculated as follows: $Average = \frac{\sum_{i=1}^{n} p_i V_i}{\sum_{i=1}^{n} V_i}$

Where n is the number of simultaneous transactions, p_i and V_i are respectively the price and the volume of transaction i. We also eliminated the last simultaneous transactions and calculate later times transaction by transaction. Finally, we eliminated the extreme values of durations. A number of observations of 13554 of a total of 28245 is obtained.

1. Estimation of the EACD (1,1) model: the exponential autoregressive conditional duration model: Our model is as follows:

$$\psi_i = \omega + \alpha_{\mathbf{j}} * \mathbf{X}_{\mathbf{i-j}} + \beta * \psi_{\mathbf{i-1}}$$

In this section we will consider the Engle and Russell (1998) model of ACD (1.1). The estimation of this model requires testing the distribution of the error term. These concern the verification of the absence of autocorrelation in the residuals and squared residuals by the Ljung-Box statistic. We will also determine the nature of the distribution using the method proposed by Diebold, Gunther and Tay (1998) and also used by Bauwens, Giot, and Grammig Veredas (2000) besides Bauwens and Giot (2001). In reality, this involves analyzing the density forecast that takes into account the history of the times to determine the distribution of future durations.

$$\psi_i = \omega + \alpha * \mathbf{X}_{\mathbf{i-1}} + \beta * \psi_{\mathbf{i-1}}$$

ψ_i : conditional expectation of the time knowing his past.

X_{i-1} : the interval of time between event arrivals.

Table 1: Estimation of EACD (1,1) model.

Variables	Coefficients	Standard Error	T Student	Significativity
ω	0.1727	0.0513	3.3654	0.0072
α	0.0848	0.0083	10.1085	0.0000
β	0.7987	0.0238	33.4733	0.0000

Residuals auto- correlation

1: -0.0109618 0.0029605 -0.0081758 0.0065795 -0.0025273 0.0032429 0.0048814 -0.0064047 -0.0002657 -0.0075018 -0.0014775
12: -0.0052032 0.0034086 -0.0027731 -0.0031065 0.0049473 -0.0021928 -0.0022029 -0.0028460 -0.0031718

Ljung-Box statistic

Q(10-0) = 5.1144 significance level 0.88340304
Q(20-0) = 6.6142 significance level 0.99777171

Squared residuals auto-correlation

1: -2.46e-04 -2.44e-04 -4.04e-04 -5.29e-06 -3.62e-04 -2.49e-04 -2.48e-04 -4.05e-04 -3.62e-04 -3.42e-04 -3.76e-04
12: -3.78e-04 -2.43e-04 -3.91e-04 -3.81e-04 7.57e-05 -3.79e-04 -3.54e-04 -3.61e-04 -3.98e-04

Ljung-Box statistic

Q(10-0)= 0.0129 significance level 1
Q(20-0)= 0.0292 significance level 1

The results of the EACD (1,1) model estimation corroborate those obtained by Engle and Russell (1998) in the fact that all parameters are positive and statistically significant. Due to the positive sign of these coefficients, a long transaction generates a necessarily long period to the next period. This confirms the dependence effect (clustering).

The results also verify the constraint of sign proposed by Engle and Russell (1998). The sum ($\alpha + \beta$) which is equal to 0,881 and is less than unity implies that the expected duration is composed by two components observed and unobserved. This amount is the duration variable of the transaction.

The Ljung-Box statistic is evaluated at two levels of high delays, indicating no autocorrelation times of the model. This approves that our model is globally efficient and unbiased estimators.

2. Estimation of the WACD (1,1) model: Weibull autoregressive conditional duration:

$$\psi_i = \omega + \alpha * \mathbf{X_{i-1}} + \beta * \psi_{\mathbf{i-1}}$$

ψ_i : Conditional expectation of the time knowing his past.

X_{i-1} : The interval of time between event arrivals.

Table 2: Estimation of WACD (1,1) model.

Variables	Coefficients	Standard Error	T Student	Significativity
ω	0.1439	0.0293	4.8977	0.0000
α	0.0826	0.0095	8.6883	0.0000
β	0.7988	0.0216	36.8626	0.0000
γ	0.7952	0.01286	61.8198	0.0000

Residuals auto- correlation

1: -0.0135932 -0.0003587 -0.0102485 0.0042145 -0.0042953 0.0009638 0.0029325 -0.0075917 -0.0013800 -0.0080434 -0.0025133

12: -0.0055951 0.0028031 -0.0031125 -0.0035185 0.0050147 -0.0023380 -0.0022694 -0.0029689 -0.0033100

Ljung-Box statistic

Q(10-0)= 6.2364 significance level 0.79502688

Q(20-0)= 7.9067 significance level 0.99246712

We have assumed in this estimation that the error term follows a Weibull distribution. The estimated results of this model (WACD) are similar to the EACD one. Indeed, the coefficient of the model remains positive and significant respecting the constraint of ($\alpha + \beta$ <1). The parameter γ of the Weibull distribution is less than one; this is the reason that explains why this distribution includes the exponential, Bauwens and Giot (2001) which validates the choice of the error term. As with the EACD model, the Ljung-Box statistic for both levels, agrees that our model generates globally efficient and unbiased estimators.

In Tables 1 and 2, the temporal durations show a significant degree of persistence with coefficients for each estimate in excess of 0.79.Moreover, the stationarity conditions for each estimation, are respected ($\alpha + \beta$ <1), highlighting the impact of transaction time in the models. The auto-correlations of residuals series and autocorrelations of the series of squared residuals, calculated on the data, are significant and indicate a strong dependence on inter and intra-temporal in the arrival rate of transactions. This means that the agents use the "timing" transactions to deliver orders, transactions and volumes. In conclusion, the short durations have different impacts on the profitability of prices, futures contracts that the long durations, and they will use this variable "time of transaction." This learning will lead to herd behaviour, reflecting the state of market psychology.

3. Extensions of the duration model: We have highlighted the ability of the WACD (1,1) model to capture the autocorrelation times between prices events. We try a second time to improve this model by adding new variables in the equation of microstructure along the lines of Engle and Russell (1998) or Bauwens and Giot (2001).

Proposals for additional variables: In this section we will introduce in the WACD (1.1) modelling, four microstructure variables. These variables are as: ***The volume of transactions during the " price event " above***, the volume may reflect the quality of information transmitted through the frequency of matches, and constitute a contribution to the modelling of WACD (1.1) model. The underlying idea is that an unexpected increase of volume in the market indicates the presence of arrival information, which has implications on the process of price formation. Thus, we can assume that the relationship between "price event" and the volume is negative since the arrival of information should result in increased volatility and the number of quotations.

The number of transactions along the precedent "price duration": this variable characterizes the intensity of trade. In fact, a high number of transactions suggests the presence of informed investors in the market. Also, the depth should be much lower than insiders are active in the market. Presumably in this case that volatility is higher after periods of high levels of transaction. We are well located within the theoretical framework formulated by Easley and

O'Hara (1992), and a negative relationship should be obtained from the expected of "price duration" and the intensity of transactions.

The size of BAS at the previous price event: we can define it as a measure of liquidity. In addition, the presence of this variable is justified by the fact that it is supposed to characterize the information asymmetry. Thus, a wide BAS can mean greater uncertainty about the fundamental value of the contract, resulting in greater variability. Therefore, we can assume that there is a positive relationship between the size of the BAS and the "price event."

The price difference justifying the new price event: we can say that over the price differential, the greater the amount exchanged is important. It is estimated the conditional expectation of the "price duration" using the WACD (1,1) model which is added the lagged level of microstructure variables. This model is formulated as follows: $\psi_i = \omega + \alpha * X_{i-1} + \beta * \psi_{i-1} + \delta Var_{t-1}$

ψ_i : Conditional expectation of the time knowing his past.

X_{i-1} : the interval of time between event arrivals.

var_i : is the level of microstructure variable retained in the *th* price event .

Log likelihood function is identical to the basic model:

$$L(X_1,......,X_{N(T)}) = ^{\gamma}$$

$$\sum_{i=1}^{N(T)} \mathrm{Log}\left(\frac{\gamma}{X_i}\right) + \gamma * \mathrm{Log}\left(\frac{\Gamma(1+1/\gamma) * X_i}{\psi_i}\right) - \left(\frac{\Gamma(1+1/\gamma) * X_i}{\psi_i}\right)$$

RESULTS AND CONCLUSION

All ACD model parameters are estimated as in the seminal paper of Engle and Russell (1998), by the Quasi-Maximum Likelihood, using the procedure of Bernt-Hall-Hall-Hausman. The results of our modelling are shown in the following table:

Table 3: WACD (1,1) durations modeling between price movements of futures contracts with the introduction of microstructure variables.

Variable	Coefficient	Student statistic	significativity
	Nominal BAS		
w	12.9699	3.9519	0.0000
α	0.4443	67.5474	0.0000
β	0.4447	157.3038	0.0000
γ	0.8137	333.5505	0.0000
δ	-0.0227	-9.2693	0.0000
Q(10)	8.2262	-	-
Q(20)	9.5360	-	-
	Transaction Volume		
w	8.2125	14.2308	0.0000
α	0.4200	83.6278	0.0000
β	0.5151	207.0154	0.0000
γ	0.8183	385.1489	0.0000
δ	-7.2523	- 2.7740	0.0000
Q(10)	7.2393	-	-
Q(20)	12.2380	-	-
	The price difference justifying the new price event		
w	11.1718	26.7011	0.0000
α	0.4090	81.05594	0.0000
β	0.5315	266.1759	0.0000
γ	0.8203	360.8799	0.0000
δ	2.9792	41.4168	0.0000
Q(10)	7.8981	-	-
Q(20)	14.6979	-	-

Continue..

Transaction Number			
w	29.7968	1.6851	0.0919
α	0.2789	10.6582	0.0000
β	0.2927	27.0511	0.0000
γ	0.8083	59.8882	0.0000
δ	-24.6406	-1.5416	0.1231
Q(10)	2.5516	-	-
Q(20)	3.0854	-	-

To sum up, the results of the study show that the expected time between two price events variables significantly related to market microstructure, including the width of the BAS, volume, number of transactions and away from prices justifying the new price event.

In particular, we have shown that the width of the BAS is negatively correlated with the expected price duration. This means that the higher the price BAS is wide, the lower will be the duration between price changes. We also found a significant and negative relationship between the expected duration of price and volume of transactions.

Indeed, an increase in the intensity of market transactions, which is the result of the arrival of new information, increased liquidity and amplify price movements. A relation of the same sign was highlighted between the variable represented by the number of transactions and the expected duration of price. This corroborates the results found by Easley and O'Hara (1992). A high number of transactions presupposes the presence of informed investors in the market, where volatility increasingly high prices and an expected duration of more and weaker. Finally, we demonstrated a positive relationship, which appears to be obvious from the price difference to justify the new event for money and the expected duration of price. The present research allows us to advance the following empirical findings: first, it seems that time has become an endogenous variable that affects how to measure price changes in the concepts of liquidity and volatility. Then, the ACD model we used has shown that high durations are followed by high durations and vice versa. Agents can therefore anticipate the phases of the market where the intensity is conducive to trading. Finally, the time for money can be modelled in combination with microstructure variable. This helps to explain some behaviour of agents and thus complement the many existing methodologies based on the psychology of financial markets.

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