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# ALTERNATIVE BETA RISK ESTIMATORS IN EMERGING MARKETS: THE CASE OF TUNISIA

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## ABSTRACT

In this paper, we use the sample selectivity model to estimate the systematic risk for Tunisian stocks. This approach is applied in the case of extreme thin trading where data are censored due to the presence of zero returns. The approach is a two-step procedure: a selectivity component which deals with the discreteness in the observed data and a regression component which applies to the non-zero return data. In addition, this study compares the new beta estimate to the standard OLS beta and the Dimson Beta. The results reveal that on average, the selectivity model corrects for the general downward bias in OLS betas more suitably ten the Dimson correction. Our approach is more appropriate to deal with the presence of zero return observations associated with extreme thin trading situations in emerging markets.

**Keywords**: Censored data, asynchronous trading, thin trading.

## INTRODUCTION

The increasing globalisation of the world's financial markets has led to a greater emphasis on the pursuit of the benefits of international diversification. In turn, this has led to consideration of a broader range of capital markets as possible investment opportunities. One such alternative is the Maghreb region in order to provide an alternative source of capital to firms from traditional banking systems, Hearn (2011). Specifically, the Tunisian market has benefitted from the European Neighbourhood Policy (European Commission website, 2010) that has facilitated the attraction of foreign investments through the provision of assistance in improving regulation and corporate governance. Following its inclusion in a number of renowned emerging market benchmark indices including Morgan Stanley Capital International (MSCI), Standard and Poors and Financial Times Stock Exchange FTSE, international investor awareness of the Tunisian Stock Exchange TSE has increased. However, the determination of an appropriate risk measure for individual stocks is a key issue for investors. The Capital Asset Pricing Model (CAPM) and the corresponding systematic risk (beta) seems to be one such alternative. But according to Pereiro (2001), the use of the CAPM in the context of emerging markets is problematic given the illiquidity patterns and the small size of the markets. One alternative is to use different measures, such as that emanating from the downside risk model (D-CAPM) suggested by Estrada (2002) whose results clearly illustrate that the CAPM beta understates the risk relative to the downside risk measure.

The empirical evidence has shown that in a univariate setting, thin trading makes the standard realised variance estimator biased and inconsistent, Griffin and Oomen (2011). Several approaches were proposed to deal with this problem as data sub-sampling, Zhang et al., (2005), the kernel-based autocovariance adjustments, Barndorff- Nielsen et al., (2008a) or the pre-averaging methods to correct the variance-covariance structure, Jacod et al., (2009) and Podolskij and Vetter (2013). The additional problem of non-synchronous trading is encountred in a multivariate setting, Fisher (1966) and Epps (1979). To overcome this problem, two conceptually different approaches have been suggested in the literature. The first mitigates non-trading biases by incorporating lead and lag

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autocovariance terms into the realised covariance estimator based on synchronized returns, Scholes and Williams (1977), Dimson (1979) and Cohen et al., (1983). The second, althouhg more compicated, operates directly on the non-synchronous data and delivers unbiased covariance estimates by accumulating the cross-product of all fully and partially overlapping event-time returns as in Hayashi and Yoshida (2005).

Despite these adjustments, previous studies focusing on thin trading have mixed results. Bley (2011) find that the correction of the return series for thin trading in the emerging Gulf Cooperation Council (GCC) stock markets, as suggested by Miller et al., (1994), fails to generate substantially different results for any GCC stock market. This explanation is supported by Parameswaran (2000) who states that thin trading does not impact serial correlation. Nevertheless, Chau and al., (2014) emphasize the need for market adjustments in the Middle East and North African (MENA) countries given that these markets are generally less developed than other emerging markets and suffer from a thin-trading problem. As Lo and MacKinlay (1990) attribute the positive autocorrelation in daily stock returns to non-synchronous trading or non-trading, the authors follow Gulen and Mayhew(2000) by removing the influence of worldwide movements and potential autocorrelation associated with thin-trading. Other works of Brooks et al., (2004a, 2005a, 2005b) investigated the problem of the understatement of risk by the CAPM beta and tested whether it is a result of data censoring associated with thin trading and/or illiquidity. In the Latin American case, Brooks et al., (2004a) ; the Australian market, Brooks et al., (2005 a) and the Canadian market, Brooks et al., (2005b), thin trading has been found to introduce a censoring problem that leads to OLS estimates of beta risk being downward biased. The authors argue that this can be overcome by using sample selectivity model to estimate betas. In a related work, Bourdriga and Trabelsi (2008) find that the instrumantal variable estimator for the market model proposed by Fowler and Rork (1983) with three leads and three lags exhibits more significant results than the OLS estimates or the Dimson (1979) corrected betas but the authors retained model is not appropriate in the case of extreme thin trading.

If the issue of thin trading is confronted in developed and comparatively liquid markets such as Latin America, Australia and Canada then the problem is likely to be intensified for the emerging and illiquid market of Tunisia. Accordingly, we are motivated in the application of this study to consider the impact of censoring on individual stock betas in the Tunisian Stock Exchange (TSE).

The paper is structured as follows. Section 2 describes the characteristics of the TSE under study. Section 3 outlines the modelling framework to be used in this paper. Section 4 presents data and the empirical results. Section 5 contains concluding remarks.

**TUNISIAN STOCK MARKET, THE INSTITUTIONAL SETTING**

The Tunisian Stock Exchange (TSE) was founded in 1969, but its role and contribution to the economy is still very limited. It was reformed in 1994 according to international standards with the creation of the CMF[1] with the same role of the Securities and Exchange Commission (SEC) in the United States. A central depository for the market, the STICODEVAM[2], was also set up and all trades are processed through brokers.

To boost the listing process on the TSE, Tunisian government has proposed a wide range of fiscal measures. To illustrate, companies are offered a 15 percent in their corporate income tax for a period of five years when they list at least 30 percent of their capital in the stock exchange. Besides, all the capital gains and dividends earned by individual investors in the TSE are free of taxes if reselled after the year of acquisition. Despite all these incentives to develop the TSE, only fifty-seven companies have their stock listed in 2011. Foreigners can participate within the limits of 50 percent of the offering of a company. Above 50 percent, a central bank approval is necessary. As the market grows foreign investors are expected to play a proportionally important role. The recovering of the TSE is recovering partly to foreign investors who have bought up to 20.2 percent of the TSE capitalization in 2011.

Overall, according to the TSE 2011 annual report, the market remains small and at the end of 2011, the 10 largest stock market capitalizations on the official list of the stock exchange accounted for 58% of market Capitalization which is around 14 452 million dinars. At the same period, market capitalization as a share of GDP is 20.8 % compared to 6.54% in 1993 (World Bank Financial Structure Database 2013) but has relatively performed well in 2011with a price-to-earnings ratio reaching 16.37[3].

**MODELLING FRAMEWORK**

The presence of the zero returns in the underdeveloped tunisian market leads to censored data. Applying least squares regressions under these circumstances produce inconsistent estimates of beta, Green (2010). In this context, Blundell and Meghir (1987) propose the sample selectivity model to deal with the problem of thin trading. The model is comprised of two components: a selectivity component which deals with the discreteness in the observed data and a regression component which applies to the non-zero return data.

The following analysis relies heavily on Brooks et al. (2004a, 2005a, 2005b). In the selectivity component, it is assumed that observed data is underlined by a latent variable, labelled$z_{it}$. We assume that $z_{it}$ is determined via a regression model with explanatory variables$w_{it}$. In the current setting we choose trading volume as the explanatory variable in the selectivity component wich appeals to the literature that has investigated the stock price–volume relation, Gallant et al., (1992), Hiemstra and Jones (1994) and Karpoff (1987). If this variable exceeds some threshold value, we observe a non-zero return and the second regression component will apply to the observed data on an individual asset's returns, rit. In other words, to observe a non-zero return, we need a sufficiently large trading volume on a given day to trigger a price change.

Once a non-zero return is observed, $z_{it}> 0$, then the regression component will apply to the data. That is, for all non-zero returns the traditional market model (a regression model) applies. The binary choice component is concerned with sample selection and the regression component is concerned with modelling the (non-zero) returns data. As in Brooks et al. (2004a, 2005a, 2005b), we formally have the following:

*Selectivity components*

$$z_{it}^{*} = w_{it}'\gamma_i + \mu_{it}$$

*Where*

$$z_{it} = \begin{cases} 1 & if\ z_{it}^{*} > 0. \\ 0 & otherwise \end{cases}$$

*Equivalently*

$$z_{it} = \begin{cases} 1\ if\ non-zero\ return. \\ 0 \qquad if\ zero\ return. \end{cases}$$

This yields a discrete choice model for the zero versus the non-zero return variable, $z_{it}$ . If we assume normality for the underlying distribution then we have a probit model with $P(z_{it} = 1) = \Phi(w_{it}'\gamma_i)$ and $(z_{it} = 0) = 1 - \Phi(w_{it}'\gamma_i)$.

When we have a non-zero return, the regression component of the sample selectivity model applies ($z_{it} = 1$). For simplicity we will assume that this regression component can be specified as the traditional market model. In our case, we have $r_{it} = \alpha_i + \beta_i r_{mt} + v_{it}$ when$z_{it} = 1$.

Assuming that error terms $(\mu_{it}, v_{it})$ follow a bivariate normal distribution, our model is presented as follows:

$$E(r_{it} |\ z_{it} = 1) = \alpha_i + \beta_i r_{mt} + \theta_i \lambda(w_{it}'\gamma_i)$$

Where $\lambda(w_{it}'\gamma_i)$ is the "Inverse Mill's Ratio", IMR.

The cause of the bias and inconsistency in ordinary least squares is caused by the omission of the IMR from the regression model. We will use the two-step procedure of Heckman (1979) to estimate the model. This procedure yields an estimator that is unbiased, consistent but not fully efficient ( ). The estimation process is as follows:

- Obtaining the $\gamma_i$ estimates after running the maximum likelihood method on the probit selection equation and obtain the Inverse Mill's Ratio.
- Replace the Inverse Mill's Ratio estimate from step (1) and run the regression model$r_{it} = \alpha_i + \beta_i r_{mt} + \theta_i \lambda(w_{it}'\gamma_i)$.

We also employ the Dimson (1979) model to treat the thin trading problem caused by asynchronous trading. This is done with the inclusion of two leads and two lags of the market return. The resulting model is as follows:

$$r_{it} = \alpha_i + \beta_{i-2} r_{mt-2} + \beta_{i-1} r_{mt-1} + \beta_{i0} r_{mt} + \beta_{i1} r_{mt+1} + \beta_{i2} r_{mt+2} + e_{it}$$

The corrected beta using the Dimson (1979) approach is then $\beta_{OLS}^{Dim}$ which is equal to$\sum_{k=-2}^{k=2} \beta_{ik}$.

Thus, in this study we will compare (i) the OLS systematic risk $(\beta_{OLS})$, (ii) The Dimson corrected beta with two leads and two lags$(\beta_{OLS}^{Dim})$, (iii) The selectivity corrected beta $(\beta_{SEL})$ and (iv) the Dimson beta being corrected for censored data $(\beta_{SEL}^{Dim})$. The latter takes into account two elements of thin trading, censoring and synchronicity.

## DATA AND EMPIRICAL RESULTS

**Data and descriptive statistics:** We examine stock returns on the TSE for a period of three years from 02 January 2009 to 30 December 2011. Data relative to stock prices, financial statements and firm market equity come from the TSE electronic database. The market return is computed from Tunindex which is a value-weighted market index. We include in our sample companies whose stocks were listed for at least two years on the TSE. The total number of observations in our sample is 35,281 and the degree of censoring varies across the 47 retained companies. The lowest censoring is 8.25% (61 zero return observations out of 739 observations, SOTETEL) and the highest censoring is 95.5%. The mean level of censoring is 34.47% which is too high compared to several emerging markets. Brooks et al. (2004a) find that the mean level of censoring is 13.35% for a sample of seven Latin countries over the period 2000-2002.

Table 1. Average and high/low estimates across censoring categories.

| Panel A: Average size, number of transactions, trading volume and betas across censoring categories | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Category | Number of firms | Size (in billions) | Transac tions | Trading volume | $\beta_{OLS}$ | $\beta_{OLS}^{Dim}$ | $\beta_{SEL}$ | $\beta_{SEL}^{Dim}$ |
| 0.082≤c<0.146 | 10 | 454 | 57 | 264,636 | 0.66 | 0.42 | 0.73 | 0.49 |
| 0.146≤c<0.169 | 10 | 409 | 50 | 194,113 | 0.62 | 0.41 | 0.69 | 0.43 |
| 0.169≤c<0.251 | 9 | 279 | 45 | 120,119 | 0.37 | 0.22 | 0.46 | 0.27 |
| 0.251≤c<0.6 | 9 | 137 | 13 | 59,319 | 0.30 | 0.19 | 0.45 | 0.20 |
| 0.6≤c | 9 | 217 | 4 | 10,781 | 0.10 | 0.10 | 0.32 | 0.58 |

| Category | $\beta_{OLS}$ | | $\beta_{OLS}^{Dim}$ | | $\beta_{SEL}$ | | $\beta_{SEL}^{Dim}$ | |
|---|---|---|---|---|---|---|---|---|
| | min | max | min | max | min | max | min | max |
| Panel B: Low/High betas across censoring categories | | | | | | | | |
| 0.82≤c<0.146 | 0.378 | 0.895 | 0.098 | 0.581 | 0.435 | 0.959 | 0.074 | 0.668 |
| 0.146≤c<0.169 | 0.343 | 0.878 | 0.120 | 0.726 | 0.482 | 0.954 | 0.148 | 0.700 |
| 0.169≤c<0.251 | -0.091 | 0.747 | -0.433 | 0.622 | -0.168 | 0.868 | -0.640 | 0.783 |
| 0.251≤c<0.6 | 0.062 | 0.461 | -0.157 | 0.317 | 0.011 | 0.643 | -0.760 | 0.496 |
| 0.6≤c | -0.027 | 0.316 | -0.017 | 0.326 | -1.180 | 1.200 | -1.771 | 5.138 |

Note: This table presents the average, low and high for each of the four different beta estimates when partitioned into one of ten groups according to the degree of censoring in the data. The censoring measure (c) is defined as the proportion of the total sample period for which zero return observations are recorded for each stock.

The full sample of the TSE companies range in size from 9 million TND to 1 400 million dinars (Poulia Holding Group). The mean company size is 30 million TND and the median company size is 135 million TND. In general, company size is negatively correlated with censoring (ρ = -0.2476). The mean of the average daily trading volume is 134031 TND and the median is 114 432 TND. In general, daily trading volume is negatively correlated with censoring (ρ = -0.6483) which justifies our choice of trading volume as the explanatory variable in the selectivity component.

**Comparison of the Betas:** Four variants of beta are calculated for the 47 companies of our sample. The estimated betas are (i) the standard OLS beta; (ii) the Dimson corrected beta with two leads and two lags; (iii) the selectivity corrected beta and (iv) the Dimson beta with the selectivity correction.

The results are presented in table 1 through table 3 and grouped in three categories according to the average in (i) the censoring degree in stock returns, (2) the market value or firm size and (3) the trading volume over the period of our study. Each table reports the average, high and low estimates across five categories and the corresponding average size, number of transactions and trading volume.

The examination of the results presented in table 1 through table 3 confirms the negative relationship between censoring on one side and size, number of transactions and trading volume on the other side. Table 1 also shows that the biggest firms are less exposed to the problem of censoring which ranges between 8.2% and 14.6% for this category with an average size of 454 million TND and an average daily trading volume of 264 thousand TND. While the daily average transactions and daily average trading volume are respectively 264 thousand TND and 57 transactions for the first category (the less exposed to the censoring problem), they are respectively 11 thousand TND and 4 transactions for the category that is less exposed to the censoring problem.

Table 2. Average and high/low beta estimates across firm size categories.

| Category | Number of firms | Deg of censoring | Nombre de transactions | Vol. (milles dinars) | $\beta_{OLS}$ | $\beta_{OLS}^{Dim}$ | $\beta_{SEL}$ | $\beta_{SEL}^{Dim}$ |
|---|---|---|---|---|---|---|---|---|
| Panel A: Average size, number of transactions, trading volume and betas across size categories | | | | | | | | |
| 9≤c<49 | 10 | 0.540 | 17 | 27,342 | 0.21 | 0.15 | 0.46 | 0.69 |
| 49≤c<100 | 9 | 0.286 | 48 | 119,440 | 0.34 | 0.27 | 0.32 | 0.14 |
| 100≤c<287 | 10 | 0.334 | 35 | 122,534 | 0.46 | 0.28 | 0.60 | 0.30 |
| 287≤c<485 | 9 | 0.239 | 42 | 196,054 | 0.47 | 0.27 | 0.55 | 0.35 |
| 485≤c | 9 | 0.228 | 35 | 217,917 | 0.63 | 0.41 | 0.76 | 0.48 |
| catégorie | $\beta_{OLS}$ | | $\beta_{OLS}^{Dim}$ | | $\beta_{SEL}$ | | $\beta_{SEL}^{Dim}$ | |
| | Min | max | min | max | min | max | min | max |
| Panel B: Low/High betas across size categories | | | | | | | | |
| 9≤c<49 | 0.004 | 0.706 | -0.157 | 0.577 | 0.002 | 1.200 | -0.760 | 5.138 |
| 49≤c<100 | -0.091 | 0.674 | -0.103 | 0.707 | -1.180 | 0.832 | -1.771 | 0.783 |
| 100≤c<287 | 0.059 | 0.895 | 0.023 | 0.539 | 0.329 | 0.959 | -0.612 | 0.594 |
| 287≤c<485 | 0.087 | 0.878 | -0.433 | 0.726 | 0.214 | 0.954 | -0.640 | 0.865 |
| 485≤c | 0.316 | 0.842 | 0.073 | 0.622 | 0.535 | 0.908 | 0.221 | 0.744 |

Note: This table presents the average, low and high for each of the four different beta estimates when partitioned into one of ten groups according to firm size. The firm size measure (M) is the average market value of equity (C$ million) across the total sample period for each stock.

Table 3. Average and high/low beta estimates across trading volume categories.

| Category | Number of firms | Deg of censoring | transactions | Size (in billions) | $\beta_{OLS}$ | $\beta_{OLS}^{Dim}$ | $\beta_{SEL}$ | $\beta_{SEL}^{Dim}$ |
|---|---|---|---|---|---|---|---|---|
| Panel A: Average size, number of transactions, trading volume and betas across trading volume categories | | | | | | | | |
| 1.17≤c<20.1 | 10 | 0.741 | 5 | 82.8 | 0.10 | 0.09 | 0.27 | 0.42 |
| 20.1≤c<88.75 | 9 | 0.406 | 17 | 222 | 0.24 | 0.10 | 0.40 | 0.16 |
| 88.75≤c<150.78 | 10 | 0.170 | 44 | 277 | 0.56 | 0.41 | 0.64 | 0.48 |
| 150.78≤c<239.72 | 9 | 0.169 | 45 | 388 | 0.54 | 0.34 | 0.64 | 0.42 |
| 239.72≤c | 9 | 0.137 | 65 | 582 | 0.67 | 0.44 | 0.76 | 0.49 |
| catégorie | $\beta_{OLS}$ | | $\beta_{OLS}^{Dim}$ | | $\beta_{SEL}$ | | $\beta_{SEL}^{Dim}$ | |
| | min | max | min | max | min | max | min | max |
| Panel B: Low/High betas across volume categories | | | | | | | | |
| 1.17≤c<20.1 | -0.027 | 0.231 | -0.157 | 0.326 | -1.180 | 1.200 | -1.771 | 5.138 |
| 20.1≤c<88.75 | -0.091 | 0.461 | -0.433 | 0.317 | -0.168 | 0.785 | -0.640 | 0.496 |
| 88.75≤c<150.78 | 0.171 | 0.842 | 0.160 | 0.707 | 0.243 | 0.896 | 0.276 | 0.744 |
| 150.78≤c<239.72 | 0.386 | 0.778 | 0.098 | 0.622 | 0.476 | 0.869 | 0.074 | 0.783 |
| 239.72≤c | 0.378 | 0.895 | 0.120 | 0.726 | 0.435 | 0.959 | 0.148 | 0.696 |

Note: This table presents the average, low and high for each of the four different beta estimates when partitioned into one of ten categories according to trading volume. The volume measure (V) is the average daily volume (000s) of traded shares across the total sample period for each stock.

A further examination of table 2, shows that while small firms have an average censoring of 54% and an average trading volume of 27 3424 TND, the largest ones rather have an average censoring of 22.8% and an average trading volume of 217 917 TND. In addition, table 3 shows that firms with small average daily trading volumes have a degree of censoring of 74.1% and an average size of 82.8 million TND while firms with big average daily trading volumes have a degree of censoring of 13.7% and average size of 582 million TND. These results confirm the positive relation between size and trading volume and the inverse relation between size and trading volume.

Danis and Kadlec (1994) found that for thinly traded stocks, the OLS betas are downward biased while for the frequently traded stocks they are upward biased. This infrequent trading is typical in emerging markets where many stocks have long sequences of zero returns, Ikbal and Brooks (2007). We thus compare the Dimson beta and the selectivity corrected OLS beta to the standard

OLS beta. The selectivity corrected OLS beta has resulted in adjustments for the general downward bias in the OLS beta, in all censoring, trading volume and size categories as shown in table 1, table 2 and table 3. This justifies the importance of making such a correction for the companies listed on the TSE.

The comparison between the standard OLS beta and the Dimson beta shows that the average standard beta exceeds the average Dimson beta in all categories. This is in conformity with Brooks et al., (2004a) who find the same trend for a panel of seven countries from Latin America. Accordingly, the Dimson beta is not making a full correction for the impacts of censoring. This justifies that the potential need to correct for censoring is more important than asynchronicity in the TSE.

We now consider a comparison of the average selectivity-corrected beta with the average Dimson corrected beta. Concerning the censoring categories (table 1), the firm size (table 2) and the trading volume (table 3), the average selectivity-corrected beta exceeds the Dimson beta in all categories. Correcting for censoring is likely to be more important than asynchronicity for the listed companies on the TSE.

Now we consider a comparison of the average selectivity-corrected Dimson beta with the OLS beta, the Dimson corrected beta and the selectivity corrected beta. The results are nearly the same across tables 1, 2 and 3. With regard to the censoring categories of table 1, the average selectivity-corrected Dimson beta exceeds the average Dimson beta in all categories, while the average selectivity-corrected Dimson beta is smaller than the selectivity corrected beta. The same results are found for the size categories of table 2 except for the smallest firms. Under the volume categories of table 3, the average selectivity-corrected Dimson beta exceeds the average Dimson beta in all categories, while the average selectivity-corrected Dimson beta is smaller than the selectivity corrected beta except for the category of the less traded companies.

Considering panel B of tables 1, 2 and 3 which shows the minimum/maximum for the four variants of beta, it should be noted that the difference between the extremums across the considered categories always achieves single figure betas with a maximum of 5,14. This is partly due to keeping firms with extreme censoring in our sample.

Table 4. Correlation matrix of censoring, firm size, trading volume and betas.

| | $\beta_{OLS}$ | $\beta_{OLS}^{Dim}$ | $\beta_{SEL}$ | $\beta_{SEL}^{Dim}$ | Size | *Transactions* | Trading volume | % of zeroes |
|---|---|---|---|---|---|---|---|---|
| $\beta_{OLS}$ | 1.0000 | | | | | | | |
| $\beta_{OLS}^{Dim}$ | 0.7794* | 1.0000 | | | | | | |
| $\beta_{SEL}$ | 0.6877* | 0.5732* | 1.0000 | | | | | |
| $\beta_{SEL}^{Dim}$ | 0.1408 | 0.3124* | 0.6489* | 1.0000 | | | | |
| Size | 0.4644* | 0.2715 | 0.3389* | 0.0088 | 1.0000 | | | |
| *Transactions* | 0.6206* | 0.3984* | 0.3640* | 0.0245 | 0.1045 | 1.0000 | | |
| Trading vol. | 0.6841* | 0.4520* | 0.4192* | 0.0567 | 0.5073* | 0.7562* | 1.0000 | |
| % of zeroes | -0.7473* | -0.4915* | -0.405* | 0.0501 | -0.2476 | -0.7271* | -0.648* | 1.0000 |

Note: This table presents the correlation matrix for the full sample of stocks.

**Beta correlation analysis:** Table 4 reports the correlation matrix of the various beta estimates, firm size, number of transactions, trading volume and censoring. We expect a positive relationship between the standard OLS beta estimates and both trading volume and number of transactions and a negative relationship with the degree of censoring. Table 4 shows a significant positive correlation of the OLS beta estimate with trading volume (ρ=0.6841), number of transactions (ρ=0.6206), size (ρ=0.4644) and a negative correlation with the degree of censoring (ρ=-0.7473). This reveals that OLS estimates are more likely to provide more reliable estimates for large liquid stocks.

Table 4 also shows that the selectivity-corrected Dimson beta is not statistically correlated with size, trading volume and the degree of censoring. As expected, the betas are positively correlated with each other.

In general, a comparison of our results with those obtained by Brooks et al., (2004a) for a panel of seven emerging Latin American countries reveals that the results are consistent. The authors find that OLS beta estimates increase with firm size (ρ= 0.1353) and trading volume (ρ= 0.0467) and decrease with the degree of censoring (ρ= -0.3339).

**CONCLUSION**

In this paper we have presented an alternative method of computing the beta risk estimator to deal with thin trading situations. The model was applied to the Tunisian Stock Exchange (TSE) dataset on a sample of daily data comprised of 47 companies for a period of three years (02 January 2009 to 30 December 2011). After grouping stocks in three categories according to the average in the censoring degree in stock returns, the market value or firm size and the trading volume over the period of our study, the empirical analysis has revealed that on average the selectivity-corrected betas exceed the standard OLS betas. We also found a negative relationship between censoring and size, and between censoring and trading volume and that size is positively related to trading volume. Overall, the selectivity model has been found to adjust for the presence of zero return observations associated with extreme thin trading situations. In the case of emerging markets, it is expected that beta risk is underestimated with the standard OLS approach given that these markets are generally less developed and suffer from a thin-trading problem. Our analysis corrects for the general downward bias in OLS betas and is likely to make it more effectively than the standard Dimson correction by increasing the estimated beta risk of individual securities. These results suggest that the selectivity model is more appropriate in thin trading situations.

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